SERVO CONTROL SYSTEMS 1: DC Servomechanisms

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ABSTRACT: This is one of a series of white papers on systems modelling, analysis and control, prepared by Control Systems Principles.co.uk to give insights into important principles and processes in control. In control systems there are a number of generic systems and methods which are encountered in all areas of industry and technology. These white papers aim to explain these important systems and methods in straightforward terms. The white papers describe what makes a particular type of system/method important, how it works and then demonstrates how to control it. The control demonstrations is performed using models of real systems designed by our founder - Peter Wellstead, and developed for manufacture by TQ Education and Training Ltd in their CE range of equipment. Where possible results from the real system are shown. This white paper is about the universally used 'work horse' of electro-mechanical systems– the DC servo control system or servomechanism.

1. What is a Servo Control System?

A servo control system is one of the most important and widely used forms of control system. Any machine or piece of equipment that has rotating parts will contain one or more servo control systems. The job of the control system may include:

- Maintaining the speed of a motor within certain limits, even when the load on the output of the motor might vary. This is called regulation.
- Varying the speed of a motor and load according to an externally set programme of values. This is called set point (or reference) tracking.

Our daily lives depend upon servo controllers. Anywhere that there is an electric motor there will be a servo control system to control it. Servo control is very important. The economy of the world depends upon servo control (there are other things to be sure – but stay with me on the control theme). Manufacturing industry would cease without servo systems because factory production lines could not be controlled, transportation would halt because electric traction units would fail, computers would cease because disk drives would not work properly and communications networks would fail because network servers use hard disk drives. Young people would become even more unbearable and they would complain more than they do now, because their music and games systems will not work without servo control.

Servo control systems are that important and it is vital to know about them. So pay attention and sit up straight – you are not on holiday and I am not writing this for the good of my health.

2. Modelling a Simple Servo System

Before we can control a system we must understand in mathematical terms how the system behaves without control. This is system modelling and it is a fundamental part of our work in control systems analysis. This white paper is about the simplest form of servo – the direct current (DC) position control servomechanism. It is important because, although it is the simplest form of servomechanism, it is used as the starting point for understanding all other servo systems

The basic form of a DC servo system is made of an electric motor with an output shaft that has an inertial load J on it, and friction in the bearings of the motor and load (represented by the constant b). There will be an electric drive circuit where an input voltage $u(t)$ is transformed by the motor into a torque $T(t)$ in the motor output shaft. Using systems modelling ideas for mechanical systems a torque balance can be written between the input torque from the motor and the torque required to accelerate the load and overcome friction. This is shown in the equation

$$J\ddot{\theta} + b\dot{\theta} = T(t)$$

Where θ is the angular position of the servo output shaft. The control objective is to control the shaft position θ or the shaft velocity $\dot{\theta}$ to be some desire value.

The input voltage *u(t)* is related to the torque *T(t)* by a **gain** *K* and the inertia divided by the friction coefficient is referred to as the system **time constant** τ , where $\tau = {}^{J}\!/_{b}$. So the system model becomes:

$$\tau\ddot{\theta} + \dot{\theta} = Ku(t)$$

In a practical servo system there will be additional components of the model which are important. Many of these are to do with the nonlinearities in the drive amplifier and friction in the mechanical components. The most important nonlinearities are the saturation voltage of the motor drive amplifier, the deadband in the amplifier, the so-called Coulomb friction in the rotating mechanical components and hysteresis (backlash) in any gearboxes that might be between the motor and the load. A good control system must include features to deal with these nonlinear features.

In this white paper we will concentrate on the linear parts of the servo system and only show some hints of non-linear issues. The linear part of the servo system model can be put in the transfer function form:

$$y(s) = \frac{K}{s(\tau s + 1)} u(s)$$

Where *y(s*) is the output shaft position and *u(s)* is the motor input. *K* is the system gain and τ (tau) is tthe time constant.

An important job for the control systems analyst is to know how to measure the values of the gains *K* and the time constant τ . To make it easier to follow in this case we can say that for example, the CE110 Servo Trainer has been designed to give a gain of one between the motor input and the motor speed, and an approximate gain of $K = 2$ between the measured speed and the measured shaft position. The nominal value of the time constant is 1.5. So the transfer function model can be decomposed into the transfer function from the motor input to the motor speed *v(s),* and the transfer function from the motor speed to the output shaft position.

$$v(s) = \frac{1}{(\tau s + 1)} u(s)$$
$$y(s) = \frac{K}{s} v(s)$$

Many control systems design tools use a state space representation of the system model. In servo systems the states are the velocity and position of the servo system output shaft. Rearranging the system transfer model gives the state space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & K \\ 0 & -\tau^{-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \tau^{-1} \end{bmatrix} u$$

$$\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$where: \quad x_1 = \theta = y = \text{the angular position output and}$$

$$x_2 = \dot{\theta} = v = \text{the angular velocity}$$

This is also written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{b}u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$where$$

$$\mathbf{A} = \begin{bmatrix} 0 & K \\ 0 & -\tau^{-1} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ \tau^{-1} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Also note that the servo system measured variables in the state model are the position of the shaft y (using a position encoder or potentiometer) and the velocity v (using a speed encoder).

The linear models given above are the basis of the design of servo controllers. A real servo however has non-linear components that influence its dynamic behaviour. The main nonlinearities are Coulomb friction in the moving parts and the dead zone and saturation in the motor input amplifier. This is advanced control and we will not cover it in this white paper.

3. Example of a Servo System

The figure 1 shows the CE110 Servo Trainer from TQ Education and Training Ltd. This is a classic and comprehensive representation of the servo control problem. It contains all relevant features that can be found in a practical servo system. The centre section of the system are the main hardware elements, from the left they are:

1. The inertial load
2. The speed sensor
3. An active load (in this case a generator, G)
4. The servo motor, M
5. An electric clutch and gearbox (can you see the picture of a gear system on the right?)
6. And under the gear system is the output shaft with a position sensor.

Figure 1: The CE110 Servo Mechanism Training System

The electric clutch allow the position system to be disconnected to study velocity control problems. The gearbox is included because servo mechanisms for position control very often have gearboxes to reduce speed and increase torque. The generator is included so that control under variable load can be investigated.

At the top of the front panel are electronic versions of all the nonlinear elements that can be found in real servos – these are used to teach nonlinear compensation and to understand what to look for in practical situations. We will be using the linear motor with internal load and position output through a gearbox to illustrate servo control in action. I might show some nonlinear behaviour in this white paper, but then again, I might not – it depends on how nice you are to me as I sit on this keyboard, all the time dreaming of my beautiful mountain homeland and *mein Verlobter*.

4. Servo System Controllers

There are many, many alternative controller design theories that can be used to control a servo mechanism. Possibly there are too many. Here is a list of most of the techniques:

1. Three term (PID) control
2. Velocity Feedback Control
3. Phase Lead Compensation
4. State Feedback Control
5. State Observer Implementation and Control
6. Linear Quadratic Regulator (LQR)
7. Linear Quadratic Gaussian (LQG)
8. Robust Control
9. Sliding Mode and Variable Structure Control
10. Dead Beat Control

Each of the above can be implemented as a continous time method or a digital method based on Z transforms. Also it is possible to use techniques such as fuzzy control and its variants. A bewildering choice is it not? And what is more, all of them can give an acceptable performance if designed with care and by an expert. For example, robust control potentially gives the best technical and practical results, but an expert is required to select the design factors required and to get a simple implementable controller.

In general, for different applications one particular technique would have advantages over the others in a way determined by the practical situation in which the servo is being used. This is where experience with real equipment is important. The use of real equipment such as the CE110 Servo Trainer helps enormously to discover the special features of the different techniques and to show what really happens to a servo-machine under different control designs. Simulations are necessary for testing, but they are not the real thing.

5. Example of a Servo Controller Design

I will illustrate just one possible control scheme - a state space control system. Specifically we use state feedback applied to the CE110 Servo Trainer and compare a simulation with real results. In state feedback the linear state space form of the Servo model is used.

The state feedback controller is:

$$u(t) = Kcss_1 r(t) - \mathbf{Kcss}' \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\mathit{Where}\ \mathbf{Kcss} = \begin{bmatrix} Kcss_1 \\ Kcss_2 \end{bmatrix} \quad \mathit{is}\ \text{the state feedback gain}$$

The signal *r(t)* is the reference signal or setpoint that the servo output *y(t)* is required to follow.

With state feedback the closed loop response of the system can be selected by combining the control law with the state space model and selecting the state feedback gain **Kcss** to give desirable closed loop poles which in turn give a desired transient response. Some desired closed loop pole patterns are:

- Second order system with users specified damping factor and bandwidth
- Second order Butterworth (maximally flat) pole locations with user specified bandwidth
- Second order Bessel pole locations with user specified bandwidth

Using the nominal parameter values for the CE110 Servo Trainer that were mentioned above it is possible to calculate the state feedback gains that will give any desired performance. For example, the state feedback gains that give a second order closed loop response with natural undamped frequency of 3 rads/second and damping factor of 0.5 is

$$\mathbf{Kcss} = \begin{bmatrix} 6.75 \\ 3.5 \end{bmatrix}$$

These gains are quite reasonable values (not too large) and so the desired response is probably attainable for the servo system that we are working with. To test the controller values, the controller is first tested on a simulation of the servo system. I have used the graphical control simulation and real time control tool CE2000, but any similar tool would give the same results. Then to validate the controller on the real system I used the real time interface in CE2000 to connect to the real servo system.

The results of applying the state feedback controller to a linear simulation of the Servo Trainer are shown in figure 2. Note that the response is perfect. The overshoot and period of the output signal oscillation are correct for desired closed loop damping factor and natural undamped frequency. However, the response (shown in figure 3) is not so good when we go to the real servo system and use realistic saturation voltages in the drive amplifier with deadband and Coulomb friction. There! I said that I would include some nonlinear results if you were nice, did I not? To overcome nonlinear behaviour it is necessary to experiment with the desired responses to avoid exciting the nonlinear behaviour and introduce special extra friction compensators. This goes beyond what we can do in this white paper, but I promise that we will make a white paper on friction compensation in the future – so visit our website sometimes.

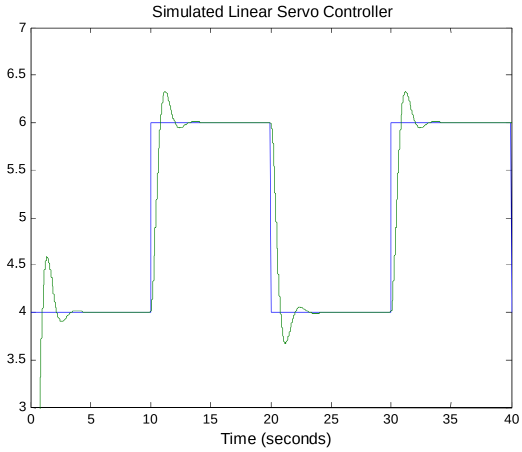

Figure 2: The Simulated Ideal Response to State Feedback

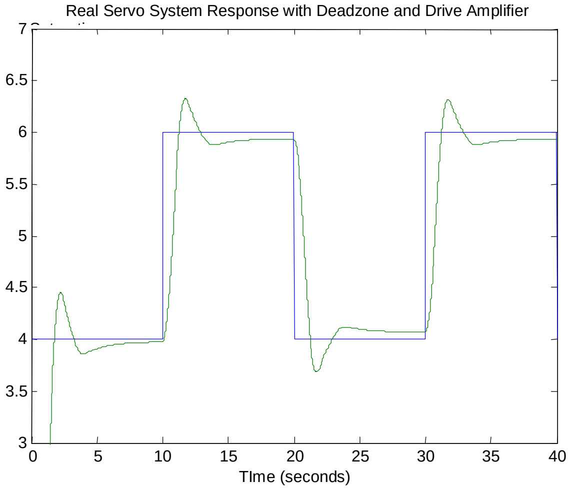

Figure 3: The Real Servo Response to State Feedback with Nonlinear Components

6. A Final Word from Elke

I hope that you have got some ideas about servo control systems from this white paper. I am sorry to say that we can not answer inquiries and questions about the detail contents of our white papers – we have much other work to do! For more information about the CE110 servo systems go to the TQ Education and Training web site, use the links on our web site or use the email info@tq.com. To learn more technical background on servo control you must read a control theory book. There are many good books on control. A book that we suggest is: *Modern Control Systems*, R.C. Dorf and R.H. Bishop, Addison Wesley.

Aufwiedersehen!

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